Error Control Coding For Meteor Burst Channels

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Abstract - The performance of several error control coding schemes for a meteor burst channel is performed via analysis and simulation. These coding strategies are compared using the probability of successful transmission of a fixed size packet through a single burst as a performance measure. The coding methods are compared via simulation for several realizations of meteor burst. It is found that based on complexity and probability of success, fixed rate convolutional codes with soft decision Viterbi decoding provide better performance.

I. Introduction

As meteors are swept up by the Earth's atmosphere, they leave a trail of ionized particles capable of reflecting radio signals. As the trail of particles diffuses and recombines, the ability of the trail to reflect signals also disappears. The useful life of these meteor trails is limited to a few tenths of a second, sometimes up to a few seconds. The random, time-varying nature of the meteor burst channel makes its analysis and use very difficult. For the purpose of analysis, an adequate model has been devised and will be presented in detail in this paper. Meteor trails are classified as overdense and underdense, based on the electron line density of the trail. In the overdense trail, signals do not penetrate the trail but reflect off the trail itself, whereas for the underdense trail signals penetrate the trail and reflect off the individual electrons. For the purpose of simplicity and mathematical convenience, only underdense trails are considered here. The model presented here closely matches that of [1]. As the underdense meteor trail disperses, the signal power reflected off the trail can be modeled as a decaying exponential given by

$$R(t) = Kq^2 e^{-\frac{t}{\tau}} \qquad (1)$$

Here R(t) is the time varying signal power, τ is the channel time constant, and q is a random variable representing the initial electron line density of the trail. K is a scaling constant. K and τ are considered deterministic and depend on the transmitter/receiver geometry, antenna gains, etc [2] [3]. Typical trail durations are from 0.2 to 1 second [4]. The bandwidth of the meteor burst channel is physically limited by multipath to be about 1Mhz. However, the FCC would typically limit the bandwidth to 20khz. The main objective in this paper will be to maximize the probability of successfully transmitting a fixed size data packet over a burst. Since we are primarily interested in relative performance, we will consider two fixed examples and study the performance of the various coding schemes over these channels. The modulation format will be coherent BPSK at 10k bits per second and the channel bandwidth is 20khz. One channel considered will have $\tau = 0.05$ seconds and a packet size of 1200 bits, and the other realization will have $\tau = 0.3$ seconds with a packet size of 2400 bits. Fixed rate BCH and Reed-Solomon codes, followed by three different possible adaptive block coding techniques will be considered. Fixed rate convolutional coding with hard decision Viterbi decoding will also be treated. Simulation packages for BCH, Reed-Solomon and convolutional codes developed for this paper will be described.

II. Simulation Package

An extensive simulation package was developed to support the research in this paper. The software was written in C++ for execution on IBM PCs and compatibles. Routines to perform the encoding and decoding for BCH, Reed-Solomon, and convolutional codes with hard decision Viterbi decoding were developed. These routines are called from various software shells written to simulate the channel in question. For this paper, the channel exhibited the exponential decay of signal to noise ratio and corresponding rise in probability of bit error characteristic in meteor burst communications. However, the channel can be quite general, involving deterministic or random channel fluctuations. Routines were written to implement the algorithms described in this paper, adaptively selecting the code rate based on the state of the channel. Some initial search algorithms were also written to locate optimum values for scaling constants and thresholds based on channel parameters for the adaptive block code algorithms considered. The search algorithms use lower bounds on the probability of successful packet transmission as described in section III and in [5].

III. Fixed Rate Linear Block Codes

Using a coarse upper bound on the probability of packet error, a search was done over all reasonable block lengths and code rates to find the optimum fixed rate code for the given channel parameters. This search was performed for Reed-Solomon codes of block length n = 7, 15, 31, and 63, as well as BCH codes of length n = 31, 63, 127, and 255. Since the packet length is in general not divisible by the number of information bits, the final block is allowed to be of a different rate to maximize the probability of success. The simple upper bound used for this search is derived as follows. For coherent BPSK, the time varying probability of bit error for our channel is given by [6]:

$$pe(t) = Q\left\{\sqrt{2\frac{R(t)}{N_0}}\right\} \qquad (2)$$

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where Q( ) is the Q function and N0 is the background noise power. To arrive at a simple error bound on the individual blocks of the message, the worst case bit error probability for the entire block is considered. This would be the error probability at the end of the block. If the block period is expressed as T_{blk}, then the worst case bit error probability for block i is expressed as:

$$pe_i = Q\left\{\sqrt{2\frac{R(iT_{blk})}{N0}}\right\} \tag{3}$$

For block i there are ki information symbols and ti correctable symbol errors. The number of correctable errors is a function of the code used. For Reed- Solomon codes of length $n = 2^m - 1$, we have

$$t_i = \frac{n - k_i}{2} \tag{4}$$

For binary BCH codes of length $n = 2^m - 1$, we have

$$mt_i \leq \frac{n - k_i}{2} \tag{5}$$

Now, for each block of the message an upper bound on the probability of block error is given by

$$be_i = \sum_{j=0}^{t_i} \binom{n}{j} pe_i^j (1 - pe_i)^{n-j} \tag{6}$$

then if there are L total blocks in the message, the lower bound on probability of packet error is given by:

$$p(success) = \prod_{i=1}^{L} (1 - be_i) \tag{7}$$

This bound for initial searches of Reed-Solomon and BCH codes for the $\tau = 0.3$ second, 2400 bit packet channel is used to find the best fixed rate block code. For Reed-Solomon codes a (63,45) code was best and for BCH codes (255,187) codes was best. These codes were then simulated and the results are shown in figure 1. For the $\tau = 0.05$ second, 1200 bit packet channel a (31,27) Reed-Solomon and a (127,120) BCH code were best. These codes were simulated and the results shown in figure 2.

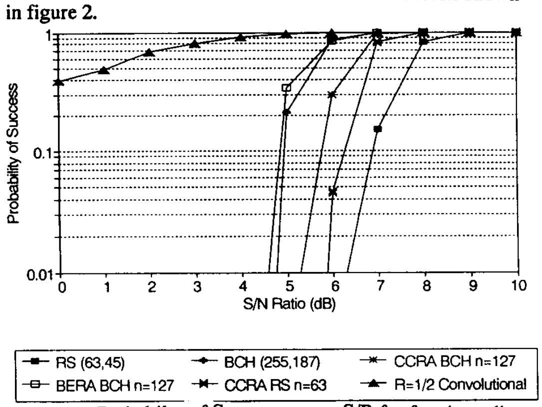

Figure 1. Probabilty of Success versus S/R for for six coding schemes ($\tau = 0.3$, 2400 bits, coherent BPSK)

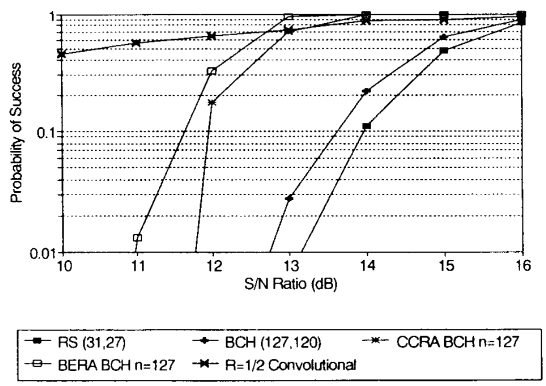

Figure 2. Probabilty of Success versus S/R for for six coding schemes ($\tau = 0.05$, 1200 bits, coherent BPSK)

IV. Adaptive Block Codes

Several adaptive coding strategies are next considered. The first and second methods considered select the number of information symbols ki for each block based on the signal to noise ratio during that block. The signal to noise ratio of the link would be available to the transmitter in a protocol where the receiver sends a probe signal. The signal to noise ratio could be determined at the transmitter by the level of the probe signal. The actual selection criteria for ki is what differs between the first and second method. The third was introduced by Pursley and Sandberg [5], which actually precomputes the optimum number of blocks, L, and the set of k's for a given initial signal to noise ratio, channel time constant, modulation format, and type of code used.

IV-A. Channel Capacity Rate Adapting

The channel capacity rate adapting (CCRA) method is designed to keep the information transmission proportional to the theoretical channel capacity while channel bandwidth is kept constant. From Shannon, the time varying channel capacity is given by [7]:

$$C(t) = B \log\left\{1 + \frac{R(t)}{N0}\right\} \tag{8}$$

where B is the channel bandwidth. To keep the number of information bits per second proportional to C(t), we have

$$k_i = \alpha B \log\left\{1 + \frac{R(iT_{blk})}{N0}\right\} \tag{9}$$

where α is a scaling constant. Presumably, α is a figure of merit for the block code used. The closer α is to 1, the closer the coding scheme approaches theoretical channel capacity. Using the packet error bound described in section III, we performed a search of BCH and Reed-Solomon codes for the $\tau = 0.05$, 1200 bit length channel and the $\tau = 0.3$, 2400 bit length channel. The search was for the best block length n and the best value for α. The optimum scale factor, α, turns out to depend on the initial signal to noise ratio and the block code used. Note that for the final block, the value of k is chosen to be the smallest such that the remaining information bits in the packet are sent. For the $\tau = 0.3$ channel, the best BCH block length was 127, while for Reed-Solomon codes, the best block length was 63. These codes were simulated and the results are shown in figure 1. For the $\tau = 0.05$ channel, the best BCH block length was 127. At this time, the search for the best Reed-Solomon is not complete. The simulation results for these codes are shown in figure 2.

IV-B. Block Error Rate Adapting

The block error rate adapting (BERA) method is designed to keep each block error probability bei below some predetermined threshold, V. To make the implementation practical, we choose to make the upper bound on block error to be below V. Therefore, the number of information bits ki of each block is chosen such that

$$\sum_{j=0}^{t_i}\binom{n}{j} pe_i^j(1-pe_i)^{n-j} < V$$

Again, the bound described in section III was used to search for the best block length and threshold V for BCH and Reed-Solomon codes. It is not surprising that the optimum threshold V increases as the initial signal to noise ratio increases. For the $\tau = 0.3$ channel, the best BCH block length was 127. The Reed-Solomon code did not perform well and was excluded. These codes were simulated and the results are shown in figure 1. For the $\tau = 0.05$ channel, the best BCH block length was 127, while the search for a suitable Reed-Solomon was unsuccessful. The simulation results for these codes are shown in figure 2.

IV-C. The Pursley-Sandberg Algorithm

Pursley and Sandberg [6] developed an iterative algorithm to compute the optimum number of blocks and number of information symbols for each block based on the properties of the probability of block error for singly extended Reed-Solomon codes. The property is that the log of bei as a function of the number of information symbols is concave. That is, log(bei(k)) is a concave function of k. In their paper, Pursley and Sandberg used a n = 32 singly extended Reed-Solomon code, noncoherent 32-ary FSK signaling at 3200 symbols per second. Their packet length was 700 bits. They recorded probability of success for channels with $\tau = 0.1$ and $\tau = 0.5$ seconds. Using the same channel configuration, we compared the CCRA and BERA methods described in sections IV-A and IV-B to the Pursley-Sandberg method. The results are shown in figures 3 and 4 and tables 1 through 6.

Algorithm	P(success)	L	k_i, i=1, ..., L
Pursley-Sandberg	0.0018	12	18,16,16,14,14,12,12,10,8,8,6,6
CCRA	0.0003	12	13,13,13,13,13,12,12,12,12,12, 12,4
BERA	0.0015	12	18,18,16,16,14,12,12,10,8,8,6,2

Table 1. Packet configuration, $\tau = 0.5$, R(0)/N0 = 1 db

Algorithm	P(success)	L	k_i, i=1, ..., L
Pursley-Sandberg	0.1002	12	18,18,16,14,14,12,12,10,8,8,6,4
CCRA	0.0289	12	13,13,13,13,13,12,12,12,12,12, 12,4
BERA	0.0706	10	20,18,18,16,14,14,12,10,10,8

Table 2. Packet Configuration, τ= 0.5, R(0)/N0 = 1.5 db

Algorithm	P(success)	L	k_i, i=1, ..., L
Pursley-Sandberg	0.5649	13	18,18,16,14,14,12,12,10,8,6,6, 4,2
CCRA	0.3007	10	16,15,15,15,15,15,14,14,14,8
BERA	0.5468	12	18,18,16,16,14,12,12,10,8,8,6,2

Table 3. Packet Configuration, τ= 0.5, R(0)/N0 = 2 db

Algorithm	P(success)	L	k_i, i=1, ..., L
Pursley-Sandberg	0.9934	13	18,18,16,14,14,12,12,10,8,6,6, 4,2
CCRA	0.9573	10	15,15,15,15,15,15,14,14,14,10
BERA	0.9804	9	20,20,18,16,16,14,14,12,10

Table 4. Packet Configuration, τ= 0.5, R(0)/N0 = 3 db

Algorithm	P(success)	L	k_i, i=1, ..., L
Pursley-Sandberg	0.0609	7	30,28,26,22,16,12,6
CCRA	0.0107	6	29,28,26,25,23,10
BERA	0.0493	7	30,30,28,22,18,10,2

Table 5. Packet Configuration, τ= 0.1, R(0)/N0 = 4.6 db

Algorithm	P(success)	L	k_i, i=1, ..., L
Pursley-Sandberg	0.9997	7	30,28,26,22,18,12,4
CCRA	0.9875	6	26,25,24,22,17
BERA	0.9995	7	30,28,26,24,18,12,2

Table 6. Packet Configuration, τ= 0.1, R(0)/N0 = 7 db

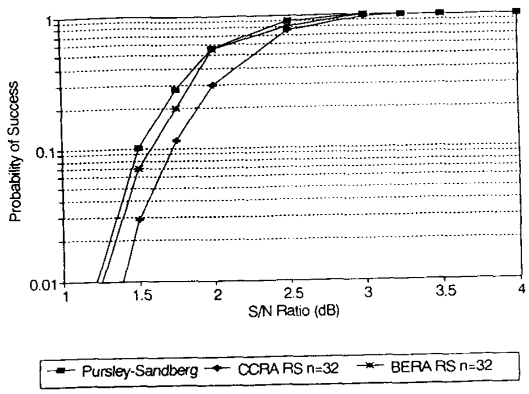

Figure 3. Probabilty of Success versus S/R for for three coding schemes ($\tau = 0.5$, 700 bits, noncoherent 32 FSK)

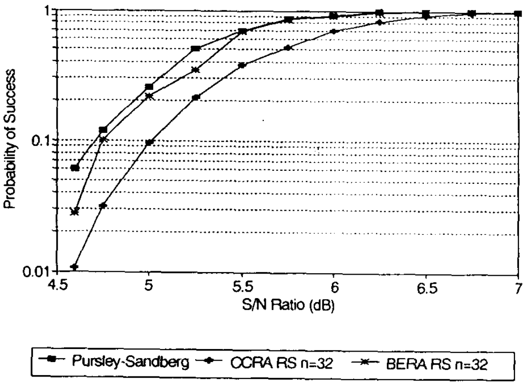

Figure 4. Probabilty of Success versus S/R for for three coding schemes ($\tau = 0.1$, 700 bits, noncoherent 32 FSK)

IV-D. Summary of Adaptive Bock Codes

The Pursley-Sandberg method is the optimum adaptive block coding scheme in terms of probability of success for the assumed channel model. However, since the number of blocks L and the number of information symbols per block are precomputed at the beginning of the burst, any deviation in the actual channel from the model used to compute the L and k's results in suboptimum performance. With the methods of sections IV-A and IV-B, a bit more robustness to channel fluctuations is inherent. The probability of success recorded in this paper were based on the model of equation (1). However, since the actual value of ki depends on the signal to noise ratio during that block, deviations from the model of equation 1 may be adapted to. In addition, the algorithm to compute the L and k's is considerably more complex than either the CCRA or BERA methods.

V. Convolutional Coding

A binary convolutional code of rate 1/2 and constraint length 7 was simulated as well. In this case the packet was padded at the end with a number of zeroes equal to the decoding depth to flush the data from the decoder. The results are shown in figures 1 and 2.

V. Conclusion

Although the simulations performed for convolutional codes with hard decision Viterbi decoding were limited to rate 1/2, k=7, it is clear that in general they would be a better choice. Only for small channel time constants with high initial signal to noise ratios did the adaptive block codes outperform the convolutional code. However, perhaps a rate 2/3 or 3/4 code would perform better than the block codes even in this situation. Also, we have not considered the coding gain advantage of soft decision decoding, widely available on many commercial convolutional decoding chips, but impractical for block codes. In light of these issues and the implementation ease of convolutional codes verses the complexity of adaptive block codes makes convolutional coding a good choice for meteor burst communications systems.

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